

Control

13/10/2015

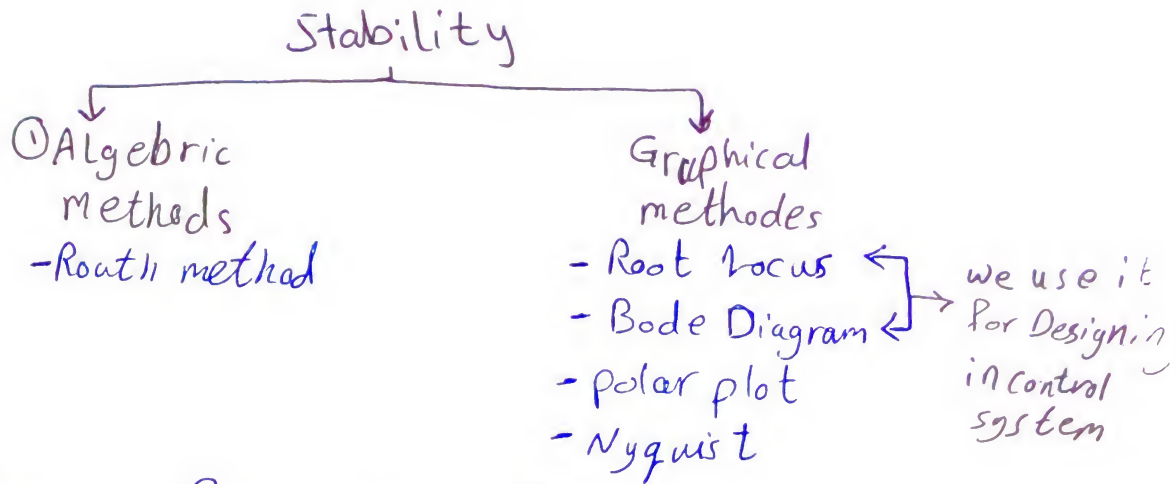
الشيخ

د. محمد عرفة

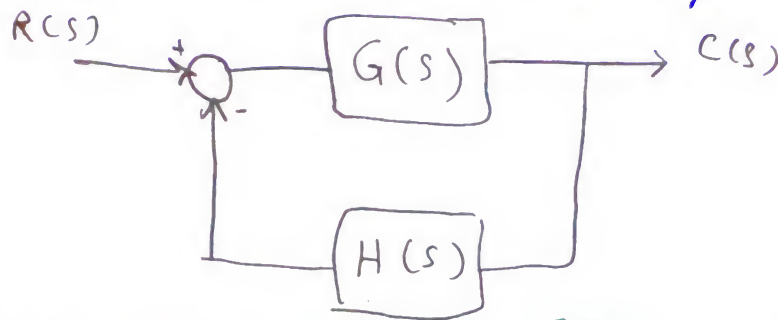
صفحة [2]

Root Locus:-

الحل الهندسي للجذور



Root locus: The locus of the roots of ch. eqn (poles)



C.L.T.F = closed loop T.F. = $\frac{G(s)}{1+GH(s)}$

O.L.T.F = open loop T.F. = $GH(s)$

ch. eqn $\Rightarrow 1 + GH(s) = 0 \Rightarrow$

① the roots of ch. equation

② = system poles

③ = eigen values

بيان جذور المعادلة

Ex: ch. eqn: $1 + GH(s) = 0$

$s^3 + 3s^2 + 4s + k = 0$

Find the range of k that make the system stable

\Rightarrow Turn over

①

تعريف آخر

Root locus:-

The locus of the roots of ch. eqn (Poles) that depend on variable parameter (K) that takes positive values ($0 \rightarrow \infty$)

Ex1 given o.l.t.f. = $\frac{K}{s(s+1)}$

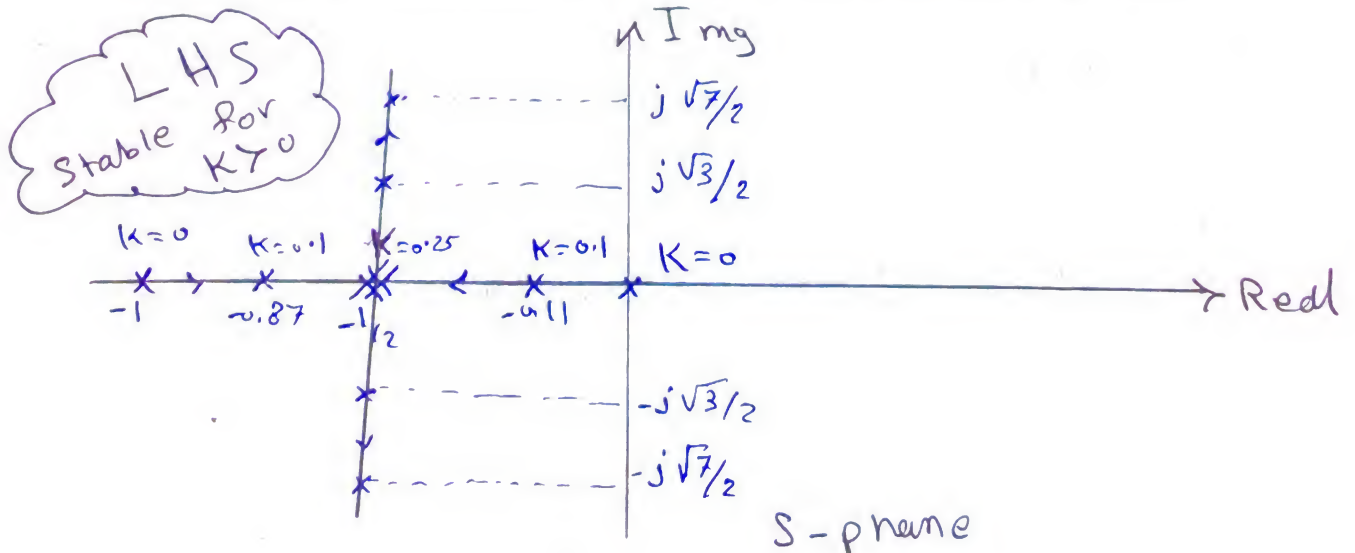
Solution: ch. eqn $\Rightarrow 1 + G H(s) = 0$

$$1 + \frac{K}{s(s+1)} = 0 \Rightarrow s(s+1) + K = 0$$

$$\boxed{s^2 + s + K} \text{ ch. eqn}$$

$$s_{1,2} = \frac{-1 \pm \sqrt{1-4K}}{2}$$

K	0	0.1	0.25	1	2
$s_{1,2}$	0, -1	-0.11, -0.88	-1/2, -1/2	-1/2 ± j√3/2	-1/2 ± j√7/2



الطريقة التالية تقريبية للاستبعاد
عن كثرة الحسابات

⇒ Turn over

Ex: $G H(s) = \frac{K(s+1)}{s(s+3)(s+4)}$

Draw the root locus and find the range of K that make the system stable

Solution:

لازم ا.ل.ت.ف. يكون حقيقي

Step ① O.L. poles \Rightarrow 3 poles \Rightarrow 0, -3, -4

O.L. Zeros \Rightarrow 1 zero \Rightarrow -1

Step ② s-plane:

Poles $\rightarrow x$

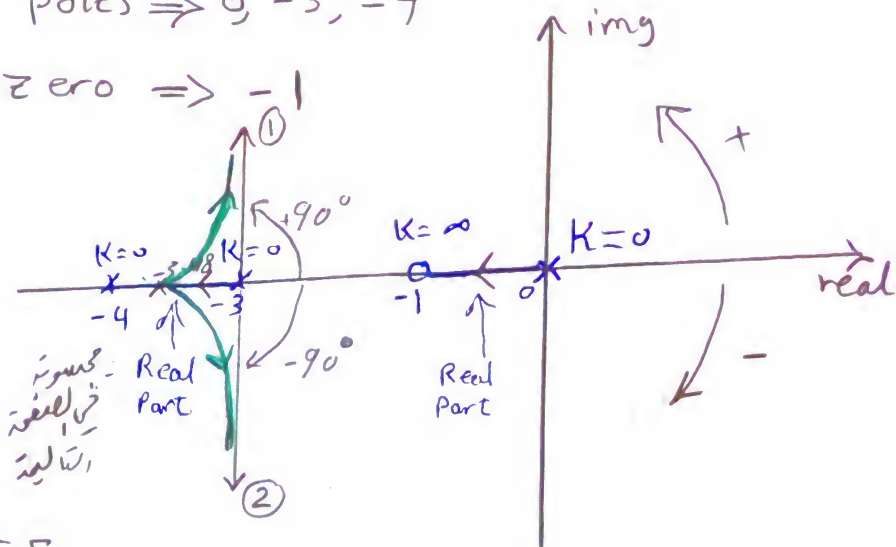
Zero $\rightarrow o$

1 Poles $\Rightarrow x$ at $K=0$

end at zero $\Rightarrow K=\infty$

at $K=0 \Rightarrow$

\Rightarrow a.l.t.f. = c.l.t.f



Step ③ Real Part:

① $0 \rightarrow -1$, ② $-3 \rightarrow -4$

Step ④ Breaking Point:

حيث نقطة تقاطع على المحاور الحقيقي
تكون فيها ا.ل. اكبر او اصغر
ما يمكن

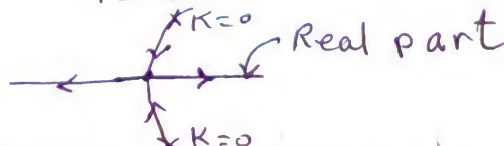
① Break away Point
نقطة هروب

K_{max}



② Break in Point
نقطة تلاق

K_{min}



Ch. equation = $1 + G H(s) = 0$ = root locus eqn

$$G H(s) = -1$$

$$\frac{K(s+1)}{s(s+3)(s+4)} = -1 \Rightarrow K = - \left[\frac{s(s+3)(s+4)}{(s+1)} \right]$$

$$\frac{dK}{ds} = 0$$

$$-\left[\frac{(s+1)(3s^2+14s+12) - (s^3+7s^2+12s)(1)}{(s+1)^2} \right] = 0$$

$$(s+1)(3s^2+14s+12) - (s^3+7s^2+12s) = 0$$

$$s^3+5s^2+7s+6=0 \Rightarrow s_{1,2,3} = -3.48, -0.75 \pm j1.07$$

\therefore Breaking Point at $s_b = -3.48$

$$K_b \Big|_{s_b = -3.48} = - \left[\frac{s(s+3)(s+4)}{(s+1)} \right] = 0.35$$

breaking point at -3.48 , $K_b = 0.35$

$K_b \rightarrow$ at breaking point, repelling poles
* two poles at the same point

Step ⑤ Asymptotes:

$$\textcircled{1} \text{ no. of asympt} = n - m$$

\uparrow no. of poles \uparrow no. of zeros

$$\textcircled{2} \sigma_c = \frac{\sum \text{Poles} - \sum \text{Zeros}}{n - m}$$

center \uparrow
of asymptotes

$$\sigma_c = \frac{(0-3-4) - (-1)}{2} = \frac{-7+1}{2} = \frac{-6}{2} = -3$$

$$\textcircled{3} \theta = \frac{(2L+1)180}{n-m}, \quad L = 0, 1, 2, 3, 4 \quad \leftarrow \text{کرر بعد از خطوط}$$

$$L=0 \Rightarrow \theta_1 = +90^\circ$$

$$L=1 \Rightarrow \theta_2 = 270^\circ = -90^\circ$$

the system is stable for all $K > 0$

- you can check by routh array.

Ch. eqn: $1 + GH(s) = 0$

$$\Rightarrow 1 + \frac{K(s+1)}{s(s+3)(s+4)} = 0$$

$$s(s+3)(s+4) + K(s+1) = 0$$

$$s^3 + 7s^2 + 12s + Ks + K = 0$$

$$s^3 + 7s^2 + (12+K)s + K = 0 \Leftarrow \text{Ch. eqn}$$

Routh array:

$$\begin{array}{c|cc} s^3 & 1 & 12+K \\ s^2 & 7 & K \\ s^1 & \frac{7(12+K)-K}{7} & \\ s^0 & K & \end{array}$$

$$\begin{aligned} 7(12+K) - K &> 0 \quad (1) \Rightarrow 84 + 7K - K > 0 \\ 6K &> -84 \\ K &> -14 \end{aligned}$$

From (1) and (2)

\Rightarrow The range for stability $K > 0$

Ex 2: $GH(s) = \frac{K}{s(s+2)(s+5)}$

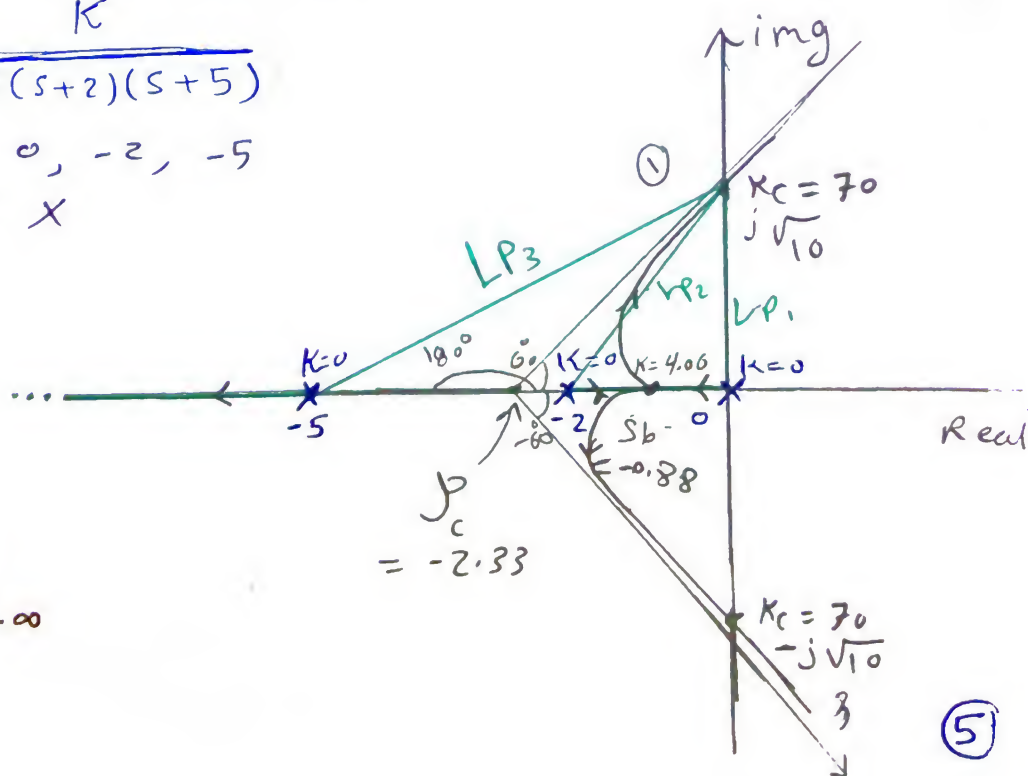
(1) O.L. poles $\Rightarrow 0, -2, -5$
O.L. zeros $\Rightarrow \times$

(2) s-plane

Pole $\rightarrow \times$

Zero $\rightarrow \circ$

(3) Real Part $\begin{cases} 0 \rightarrow -2 \\ -5 \rightarrow -\infty \end{cases}$



④ Breaking Point

$$\text{ch. eq} \Rightarrow 1 + GH(s) = 0$$

$$GH(s) = -1$$

$$\frac{K}{s(s+2)(s+5)} = -1 \Rightarrow K = -[s(s+2)(s+5)]$$

$$\frac{dK}{ds} = -[3s^2 + 14s + 10] = 0$$

$$3s^2 + 14s + 10 = 0$$

$$\Rightarrow s_{1,2} = -0.88, -3.78$$

Accepted neglected

$$s_b = -0.88$$

$$K_b \Big|_{s_b = -0.88} = -[s(s+2)(s+5)]$$

$$\Rightarrow K_b = 4.06$$

⑤ Asymptotes:

$$\text{① no. of asymptotes} = n - m = 3 - 0 = 3$$

$$\text{② } p_c = \frac{\sum \text{poles} - \sum \text{zeros}}{n - m} = \frac{(0 - 2 - 5) - (0)}{3}$$

$$p_c = \frac{-7}{3} = -2.33$$

$$\text{③ } \theta = \frac{(2L+1) \cdot 180^\circ}{n - m}$$

$n < m$
3

$$L = 0 \Rightarrow \theta_1 = 60$$

$$L = 1 \Rightarrow \theta_2 = 180$$

$$L = 2 \Rightarrow \theta_3 = 300 = -60$$

⑥ at imaginary axis \Rightarrow asymptotes مع asymptotes
 المحاور التخيلية (يجب قيمته عند أي K عند تقاطع)

ch. eqn $\Rightarrow 1 + GH(s) = 0$

$$S(S+2)(S+5) + K = 0$$

$$S^3 + 7S^2 + 10S + K = 0$$

using Routh array

$A(s)$	S^3	1	10	
\Rightarrow	S^2	7	K	
$\xrightarrow{\text{نصف فوري}}$	S^1	$\frac{70-K}{7}$	> 0 ①	$\Rightarrow K < 70$
	S^0	K	> 0 ②	$\Rightarrow K > 0$

the range for stability

$$0 < K < 70$$

at $K = 70$

the row $S^1 \Rightarrow$ has zero values

$$K_c = 70$$

critical

$$A(s) = 7s^2 + K_c = 0$$

$$7s^2 + 70 = 0 \Rightarrow s^2 = -10 \Rightarrow s = \pm j\sqrt{10}$$

K at point S_0 located at the root locus:

$$K \Big|_{S_0} = \frac{\prod \text{Poles}}{\prod \text{Zeros}}$$

$$K|_{s_0} = \frac{L_{P_1} \cdot L_{P_2} \cdot L_{P_3}}{1}$$

مقياس كفاءة كانت على

لوصل كل نقطة بكل pole وبقب أقال كقطب
 $L_{P_i} = \dots$
 $L_{Z_i} = \dots$
 $\prod_{Z_{005}} = 1$ رفع

Report :

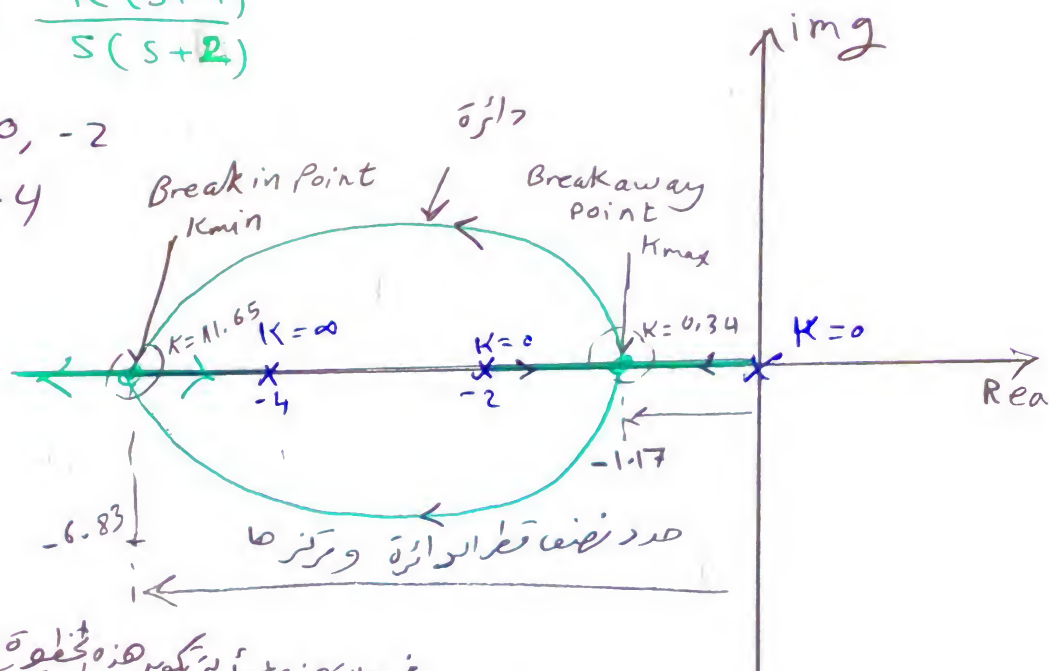
أثبت النظام السابق
 صدارة : ابدأ بال Ch. eqn

EX3:- $GH(s) = \frac{K(s+4)}{s(s+2)}$

① O. L. Poles $\Rightarrow 0, -2$
 O. L. Zeros $\Rightarrow -4$

② s-plane

Pole $\rightarrow x$
 Zero $\rightarrow o$



③ Real Part

① $0 \rightarrow -2$
 $-4 \rightarrow -\infty$

④ Asymptotes

① No. of asy. = $n - m = 2 - 1 = 1$

② $\sigma_c = \frac{(0 - 2) - (-4)}{1} = 2$

③ $\theta = \frac{(2L + 1) 180}{n - m}$

$L=0 \Rightarrow \theta = 180^\circ$

④ Breaking Points

Ch. eqn : $1 + GH(s) = 0 \Rightarrow GH(s) = -1$

$$\frac{K(s+4)}{s(s+2)} = -1$$

$$K = - \left[\frac{s(s+2)}{(s+4)} \right]$$

$$\frac{dK}{ds} = - \left[\frac{(s+4)(2s+2) - (s^2+2s)(1)}{(s+4)^2} \right]$$

= 0

$$\therefore (S+4)(2S+2) - (S^2 + 2S) = 0$$

$$S^2 + 8S + 8 = 0$$

$$S_{1,2} = -1.17 \text{ و } -6.83$$

Break away

Break in

Breaking points

$$\textcircled{1} S_{b1} = -1.17 \Rightarrow K_{S_{b1}} = 0.34$$

$$\textcircled{2} S_{b2} = -6.83 \Rightarrow K_{S_{b2}} = 11.65$$

for the circle

$$r = \frac{(6.83 - 1.17)}{2} = 2.83$$

نصف قطر الدائرة

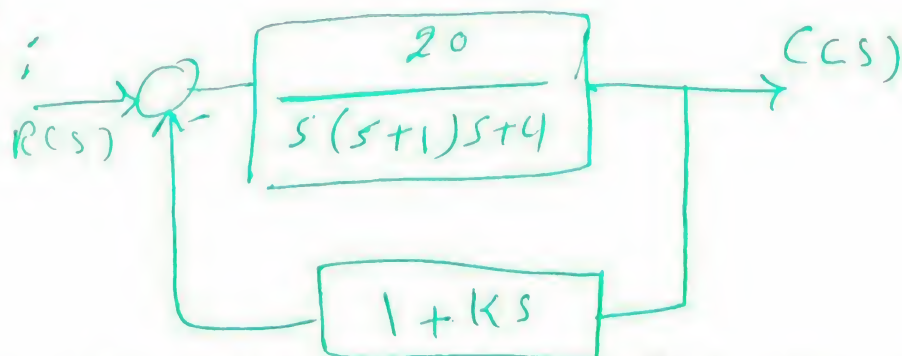
$$C_{\text{center}} = -2.83 - 1.17 = -4$$

هذا النوع هو مستقر لأن المطلوب بدور

بمقياس (أي لا يكون هناك قطبان أفرى)

system is stable for all $K > 0$

Report:



Sketch the root locus and find the range of K for stability

ملاحظة: إذا كان لازم أن يكون مشد مجبوبة مع
رغم أن مقبولة في رقم

$$GH(s) = \frac{(K+3)s}{(s+3)(s+5)} \quad \Leftarrow \begin{matrix} \text{أربع للصورة} \\ 1 + GH(s) \end{matrix}$$

و كما لا تقل أن K في طرف لومدها

$$GH(s) = \frac{(20K)(s+4)}{(s+3)(s+5)}$$

بدل أن K 20K بتغير آخر مثل (K') وهذا لأن

ويبقى الأعلام على K' أنها 20K